air in the hypersonic slender body boundary layer," General Electric Co. Rept. R64SD22 (August 1964).

[31] Knystautas, R., "The growth of the turbulent inner wake behind a 3 inch diameter sphere," Canadian Armament Research and Development Establishment TR 488/64 (February 1964).

[32] Primich, R. and Steinberg, M., "A broad survey of free-flight range measurements from the flow about spheres and cones," General Motors Corp., Defense Research Labs. TR 63-224 (September 1963).

[33] Washburn, W. K., Goldburg, A., and Melcher, B. W., "Hypersonic cone wake velocities obtained from streak pictures," AIAA J. 2, 1465–1467 (1964).

[34] Labitt, M., "The measurement of electron density in the wake of a hypervelocity pellet over a six-magnitude range," Lincoln Laboratory, Massachusetts Institute of Technology (April 1963).

[35] Hall, J. G., Eschenroder, A., and Marrone, P., "Blunt-nose inviscid air flows with coupled non-equilibrium processes," J. Aerospace Sci. 29, 1038–1051 (1962).

[36] Nawrocki, P., "Reaction rates," Geophysics Corp. of America Rept. 61-2-A (1961).

[37] Lin, S. C. and Teare, J. D., "Rate of ionization behind shock waves in air," Avco/Everett Research Rept. 115 (1962).

[38] Chanin, L., Phelps, A., and Biondi, A., "Measurements of the attachment of low-energy electrons to oxygen molecules," Phys. Rev. 128, 219 (1962).

MAY 1965 AIAA JOURNAL VOL. 3, NO. 5

Measured Transition from Laminar to Turbulent Flow and Subsequent Growth of Turbulent Wakes

W. G. Clay,* M. Labitt,† and R. E. Slattery‡

Lincoln Laboratory,§ Massachusetts Institute of Technology, Lexington, Mass.

Turbulent transition distances and growth of turbulent wakes behind $\frac{3}{16}$-in. copper-clad aluminum spheres traveling through rarefied air at 18,000 to 20,000 fps are presented. The data are taken by two independent methods: twin schlieren systems (optical) and a UHF microwave cavity (electronic). The turbulent transition distances are measured to several thousands of sphere diameters at the lower pressures by both methods. Turbulent wake widths are also determined by both techniques and the $\frac{1}{3}$ power growth rate is confirmed for these velocities.

Introduction

OVER the past several years, various measurements of the transition from laminar to turbulent flow in the wake behind various hypervelocity bodies have been reported.[1–3] These measurements, in most cases of direct observation, have been limited to transition occurring quite close to the body, since they have depended either on schlieren or shadowgraph techniques of limited sensitivity or on the observation of rapidly decaying radiation. Indeed, a body of opinion has grown up which considers that transition never occurs further behind a body than approximately 100 body diameters.[4] In order to investigate this problem and several others, new microwave techniques for measuring the width of the electron wake have been developed and combined with improvements in schlieren sensitivity at Lincoln Laboratory. These techniques have permitted the measurement of transition several thousands of body diameters behind hyper-velocity spheres in the ballistic range. In addition, these techniques have been used to measure wake growth by observation of the electron wake and the gas density gradients. The effects of projectile velocity on both of these phenomena have also been investigated and are reported in the paper.

This paper is divided into two parts: techniques and results. In the former, the schlieren and microwave techniques are described; in the latter, the experimental results are presented and discussed.

Measurement Techniques

The schlieren results have been accumulated by utilizing two spark schlieren systems of very high sensitivity which are physically side by side in the same experimental chamber at our larger ballistic range. The systems are completely independent, and the pictures that they take are independently timed from a common trigger pulse that originates when the projectile is in the center of the first schlieren system.

Figure 1 is a sketch of one of the systems being used. It is a double-pass system similar to those previously described by two of the authors[5] but utilizing line spark sources. Each mirror is a 12-in.-diam first-surface sphere of 12-ft radius of curvature. The sensitivity of a schlieren system depends on the focal length of the mirror employed, the number of times the light rays pass through the disturbance being observed, and the size of the light source.[6] Generally, the size of the light source is limited by the requirement that sufficient light be provided to take spark photographs, since the emissivity of the spark plasma remains roughly constant as the size is changed. However, in the case of knife-edge schlieren (as opposed to dot or other nonlinear schlieren obstacles) the results are unaffected by extending the light source parallel to the knife edge. Thus, one may narrow the slit, obtaining improved sensitivity of a smaller source, while the total light intensity is held to the required value by making the source

Received September 8, 1964. The authors are indebted to W. M. Kornegay of Lincoln Laboratory for his helpful suggestions and discussions. We also wish to thank A. P. Ferdinand and J. F. X. Goodwin for their assistance in obtaining and reducing the data reported here.

* Staff Member, Re-Entry Signature Studies Group. Member AIAA.

† Staff Member, PRESS Field Sites.

‡ Associate Leader, Re-Entry Signature Studies Group. Member AIAA.

§ Operated with support from the United States Advanced Research Projects Agency.

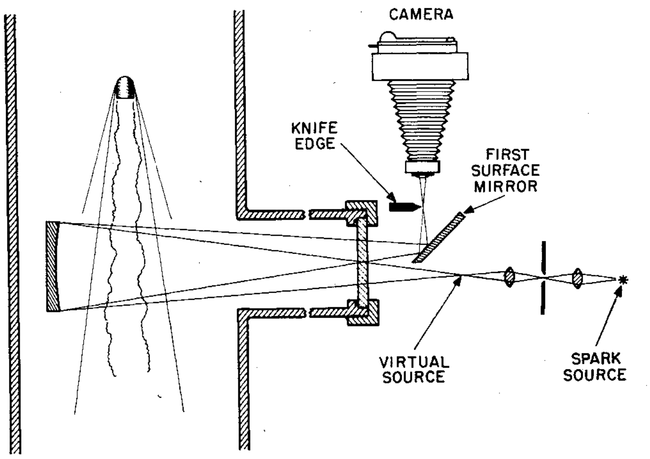


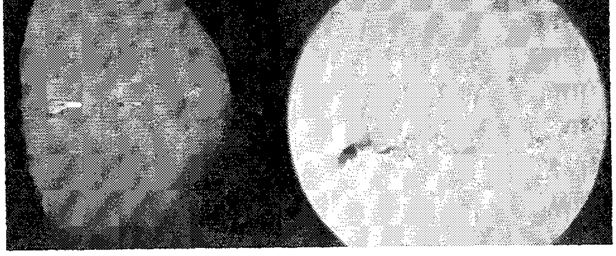
Fig. 1 Sketch representation of the schlieren system.

fairly long. In this work the authors have utilized a spark designed by A. P. Ferdinand of Lincoln Laboratory. It is characterized by great spacial stability, high and uniform light output, and relatively short duration (0.25 μsec). Geometrically, the spark is a simple plane-to-plane discharge confined to an open channel and initiated by a separate trigger electrode. The spacial stability of the spark permits it to be focused onto a slit, and the slit is then used as the schlieren light source. The slits used were all 0.080 in. long and varied in height (i.e., the direction perpendicular to the schlieren knife edge) from 0.003 to 0.020 in., depending on the sensitivity required. With this system, which is capable of extreme sensitivity, it is frequently desirable to degrade the sensitivity at higher pressures, and for this purpose the wider slit is used.

In all cases the camera is focused onto the plane of the disturbance, reducing the refraction caused by shadowgraph effects (at lower pressures shadowgraph effects are weak and can be eliminated entirely by this method). This focusing also results in much sharper schlieren photographs and, unlike shadowgraph, in no loss of contrast.

As an example of the sensitivity of the system, Fig. 2a shows the wake 6250 body diameters behind a $\frac{3}{16}$-in.-diam sphere traveling 20,000 fps. (The slit employed in taking this picture was 0.003 by 0.080 in.). In this case the wake has the "corkscrew-like" form of a subsonic turbulent wake but the general appearance is laminar. It is interesting to compare this with the subsonic wake shown in Fig. 2b.

The width of the electron wake is measured by means of a UHF microwave cavity, operating at about 440 mc/s in the TM_{010} mode. The projectile passes through the cavity, and the electric field of the cavity generates a complex dipole moment in the ionized wake (that is assumed to have a cylin-



a b

Fig. 2 Similarity in the locus of the wake 5000 body diameters behind a) a $\frac{3}{16}$-in.-diam sphere traveling 20,000 fps through 5mm Hg of air, and b) a $\frac{1}{4}$-in.-diam sphere traveling 850 fps through 20 mm Hg of air.

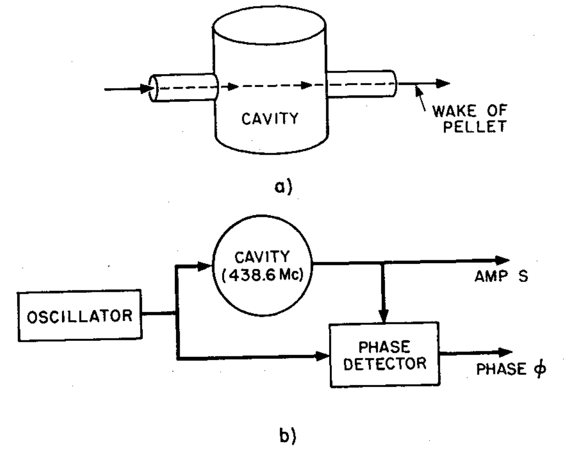


Fig. 3 Cavity and electronics used to measure electron wake width.

drical shape and a uniform electron density). Laminar flow calculations[7] and schlieren photographs bear out the assumption of sharp boundaries. The technique of measuring the dipole moment yields an average electron volume density and wake width based on the assumption of a uniform cylindrical distribution. Figures 3a and 3b show the cavity geometry and a simplified block diagram of the measuring circuitry. Since the cavity is of the order of 90 body diameters long in these experiments, the assumption of average cylindrical symmetry seems amply justified.

The details of the cavity technique have been described elsewhere,[8,9] and the process will merely be outlined here. One measures the change in phase of the transmitted signal φ and the ratio of running to initial signal amplitude S as a function of time after the passage of the pellet. The electron density n and the electron wake radius R may then be expressed as

$$n = \frac{2m\epsilon_0\omega^2}{e}\left[1 - \frac{(\nu/\omega)\sin\varphi}{\cos\varphi - S}\right] \qquad (1)$$

$$R^2 = \frac{\alpha^2 J_1^2(\alpha)\,\delta c}{4\omega\, Q_L}\;\frac{1}{\int_0^\alpha J_0^2(x)dx}\;\frac{2\cos\varphi - (1/S) - S}{\left(\frac{\omega}{\nu}\right)(S - \cos\varphi) + \sin\varphi} \qquad (2)$$

where m, ν, ϵ_0, c, and e have their usual meanings, ω is the cavity resonant frequency, α is the argument of the first zero of the Bessel function J_0, Q_L is the loaded Q of the cavity, and δ is the distance between the two flat faces of the cavity.

The validity of this technique has been checked using dielectric rods, quartz discharge tubes, and laminar wakes. Data reduction has been programed into an IBM 7094 computer, and a typical printout from an x-y plotter is shown in Fig. 4 for the wake of a $\frac{3}{16}$-in. sphere traveling through a 20-mm Hg pressure air at 20,200 fps. Data are only plotted out to 2 msec in this case. Beyond this point (ν/ω) approaches $(\cos\varphi - S)/\sin\varphi$ and the value of the radius becomes indeterminate.

In Fig. 4 the initial radius of the electron wake is found to be constant and in substantial agreement with the theoretical work of Feldman[7] for laminar wakes. This constant radius is assumed to be indicative of a laminar wake and is verified by schlieren photographs. This result disagrees with some of the theoretical assumptions made by previous authors.[10] Extrapolation of earlier work would lead one to expect a long laminar wake for this small sphere at so low a pressure.[1] This is confirmed, but at something over 1000 body diameters the wake begins to grow, and this is assumed to be the point where turbulence sets in. Beyond this point the growth rate is that which is expected for a turbulent wake [i.e., wake width/body size $\propto$ (length of trail/ body size)$^{1/3}$]. The point where the growth begins is indicated by an arrow in Fig. 4, and is assumed to be the position of laminar-to-turbulent transition. Schlieren photographs confirm the laminar nature of the region of constant width and the turbulent structure of the expanding region.

It is characteristic of this technique that the volume density of electrons is measured, as well as the wake radius. For times after the radius measurement has ceased to have physical validity, Kornegay[11] has shown that the electron density-wake area product is still meaningful. Thus a line density n_l can be computed

$$n_l = \pi R^2 n$$

that is directly comparable with the line density measured by the line density cavity technique developed by one of the authors.[8] This has been done on many shots, the projectile first traversing the line density cavity and then the width measurement cavity. These cavities are about 5 ft apart. A typical comparison of results from the two cavities is shown in Fig. 5. The results from the two cavities agree to within 20% over four or five orders of magnitude in n_l for trails as long as 25,000 body diameters. Agreement of this sort indicates the statistical reproducibility of sections of wake 90 body diameters long from the same projectile measured at different stations, and also serves as a check on internal consistency.

Earlier experiments were performed using aluminum spheres, but the data presented in this paper were all taken using copper-coated, saboted, aluminum $\frac{3}{16}$-in.-diam spheres that had been cleaned by ultrasonic agitation in ethyl alcohol. These behave like copper spheres. We have been unable to obtain any spectrographic evidence that the copper ablates down to the aluminum within our experimental distances. In all cases where electron density measurements are quoted, the experimental tanks were pumped to less than 5×10^{-5} mm Hg, and the leak rate never exceeded 2×10^{-5} mm Hg increase per minute. The impurities are felt to arise exclusively from outgassing. This conclusion is reinforced by mass analysis of the residual gas. When the precautions described previously are observed, the tanks tend to ingas rather than outgas as air is let into the tank. Filling of the tank was done with purified air from cylinders, and the impurities fraction in the experimental environment is of the order of 10^{-5}.

Results

Figure 6 is a plot of the distance from the body to the point of laminar-to-turbulent transition divided by the square root of drag area vs normalized pressure [i.e., $x_T/(C_DA)^{1/2}$ vs $P(C_DA)^{1/2}$]. It will be noted that there are two curves: a lower curve consisting of data behind bodies of various shapes traveling at less than 16,000 fps (see Ref. 1), and an

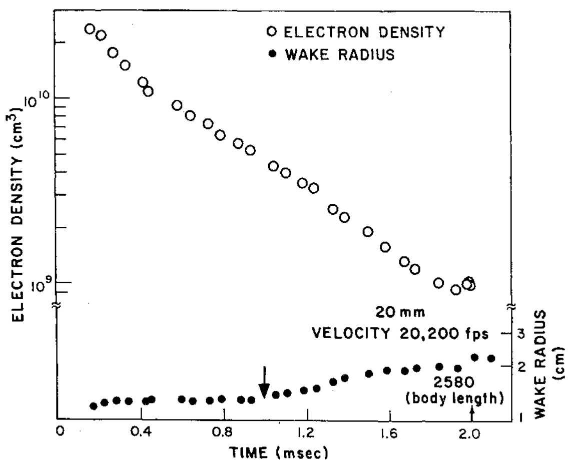


Fig. 4 Typical output of an electron wake width date measurement.

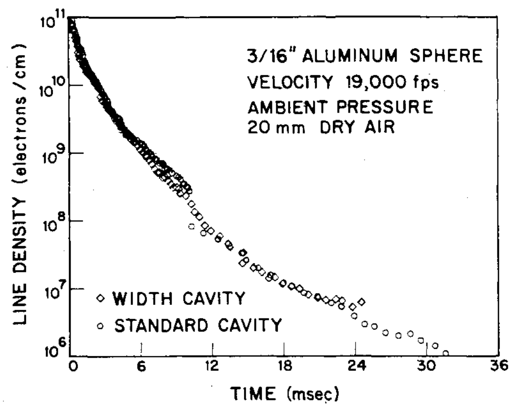


Fig. 5 Comparison of standard and width cavity results.

upper curve for spheres in the 18,000 to 20,000 fps velocity range. The data points of the upper curve in the lower pressure end consist of electron wake data (where there is a change in the growth rate of the wake) as well as schlieren data. One point from the doppler analysis of radar returns behind an 8-in. sphere fired at 22,000 fps from Wallops Island is also shown. When schlieren data were taken far behind the body, the electron wake was used to help search the trail for the transition point. Since the field of view of each of the schlieren systems is only 50 body diameters, delays were set into the individual spark sources so as to bracket the position roughly indicated by change in growth of the electron wake. In a series of shots the time between the sparks was reduced. Thus, at thousands of body diameters behind the bodies it was possible to assign regions where transition took place. When one is this far back in the wake, the transition point is not clean-cut or stable in space-time so that only regions have been assigned. The data flags in Fig. 6 are assigned in this fashion and represent the final bracketing space. The region above the curve is turbulent and below it is laminar as evidenced from a series of bracketing pictures. In this way one not only photographs the transition region, but on the same wakes one accumulates data in different ways and confirms that the two regions of the electron wake that grow differently are indeed laminar and turbulent.

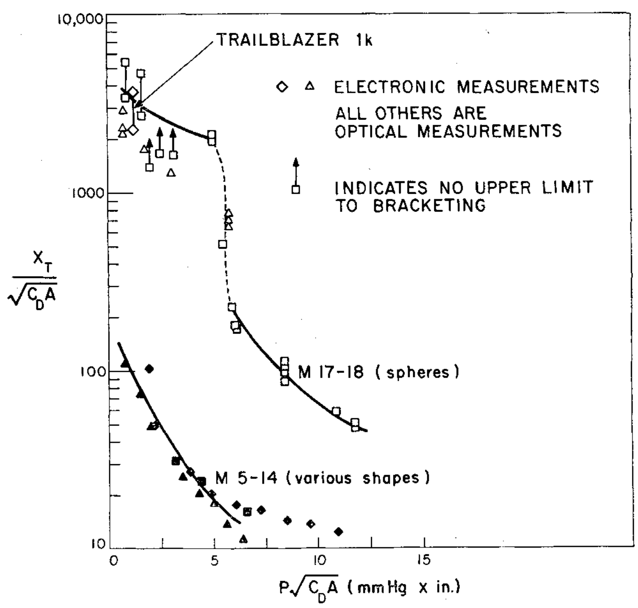


Fig. 6 Laminar to turbulent transition distances as a function of drag area, pressure, and Mach number.

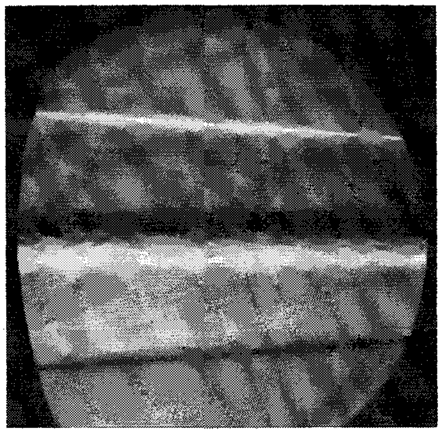


Fig. 7 Typical series of shocks originating from a transition 146 body diameters behind a $\frac{3}{16}$-in.-diam sphere traveling 18,000 fps through 60 mm of air.

Figure 6 demonstrates two new and interesting phenomena. Firstly, the distance behind the body at which transition occurs seems to be extremely velocity dependent above 16,000 fps. Secondly, it will be noted that there is apparently a break in the upper curve, where the smooth progression of transition distance away from the body with decreasing pressure appears to undergo an abrupt change. Considerable difficulty was encountered in obtaining even one data point in the region of $[x_T/(C_DA)^{1/2}]$ between about 200 and 1200 due to the very narrow range of pressures involved in the break in the curve. At higher pressures, to the right of the break, the onset of turbulence is similar to the data published previously, and when the laminar flow turns turbulent small shocks (Mach waves) are seen to originate from the turbulence (Fig. 7). Presumably these arise because the laminar wake is supersonic with respect to the surrounding inviscid region. At lower pressures, to the left of this region of apparent abrupt change in the transition curve, these shocks are no longer seen when the wake becomes turbulent. Figure 8 shows a picture of transition in the center of this region. There are no shocks coming from the wake either in the region of full transition or from the small protuberance at the left of the picture. Pictures of similar protuberances occurring closer to the body in the other pictures at this pressure do originate shock waves. These remarks concerning the Mach number of the wake core with respect to the surrounding gas are purely observational. The relationship, if any exists, between this phenomenon and the abrupt break in the transition curve is not understood.

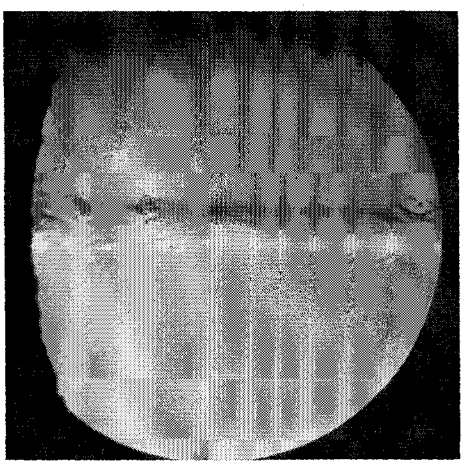


Fig. 8 A typical transition, in the case at p = 35 mm in the middle of the break in the upper curve of Fig. 6.

The authors suggest that one possible explanation for the abrupt change in the smooth behavior of the curve may be that as one goes to lower pressures for a given body size the transition to complete turbulence does not occur until the laminar wake is subsonic. At still lower pressures, where transition is quite far back in the wake, the turbulence appears different and in some ways reminiscent of subsonic wakes (Fig. 2 is an excellent example of this). In this subsonic region, the position behind the body at which turbulence can occur seems to vary within rather wide limits, and, in fact, the cavity data seems consistently lower than the schlieren measurements. In the schlieren technique one depends upon a subjective recognition by the experimenter of turbulence as it first becomes visible on a photographic plate. For these high projectile velocities, the laminar wake is initially quite hot, its density is very low, and it is possible that turbulence may occur within this isentropic region and still not be recognizable to the viewer until it bursts out of its heated core. Thus the cavity might see this growth before the eye would. This is put forward as a possibility; one might also argue conversely that the wake has had ample time to cool, and that there should be no differences between the two methods other than that expected for the 90 body diameter resolution length of the cavity. The true cause of the differences is not really understood at present.

Prior to transition the laminar wakes, as seen by schlieren and microwave cavity techniques, demonstrate a slight pressure dependence. This is shown in Fig. 9. At the higher pressures, where transition in the wake occurs close to the body, the least measure of the cavity (about 90 body diameters) prevents measurement. The curve in Fig. 9 is for a constant velocity of 18,500 fps. A velocity effect must also exist since the few points we have for laminar wakes at lower velocities (8500 fps) but the same pressure regions indicate a much narrower wake. Since we have found that laminar wakes grow very slowly, the values chosen are midway between the sphere and the transition point. The agreement between the cavity and schlieren technique is seen to be excellent.

Once the wake turns turbulent, its width may still be measured by both schlieren and cavity techniques. The points used in the schlieren curves represent individual shots, whereas the cavity technique gives continuous readings for a single shot. In Fig. 10 the growth of the turbulent wake, as measured by both of these techniques, is plotted.

In view of the excellent agreement between the cavity and schlieren techniques for the laminar wake, the authors were surprised to find the cavity data consistently and uniformly lower in the turbulent region. The measured width of the electron wake at all pressures is about 25% lower than that of the schlieren-measured wake. This may arise, in part, from the definition of width used in the turbulent case where an average width is used. However, even if one uses an extreme

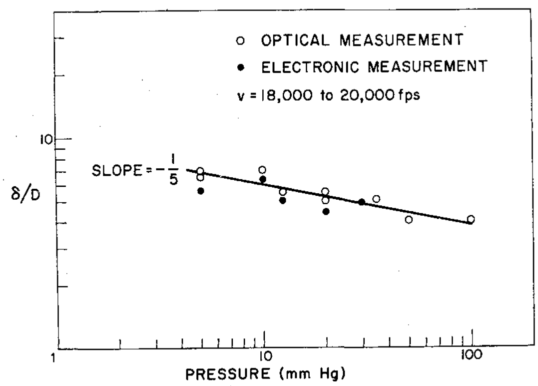


Fig. 9 Laminar wake width vs pressure.

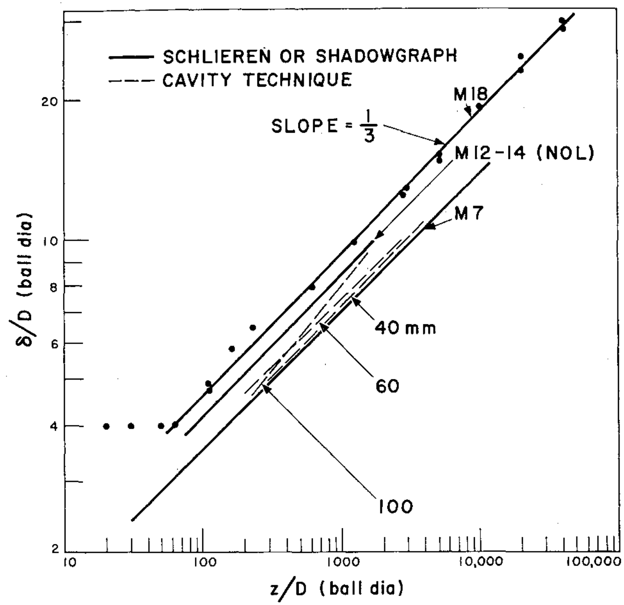


Fig. 10 Width of the turbulent wake of spheres measured different ways under several conditions.

example and plots the core of the schlieren wakes, the results still fall above the electron wake values. The reasons for this are not understood, and despite the agreement between the two techniques for the laminar wake, we are not prepared to call this a real effect at the present time. The growth rate for both cases is extremely close to the expected $\frac{1}{3}$ power.

The data from several other sources are plotted in Fig. 10, and the evidence of velocity dependence is clearly shown. At lower velocities the width of the turbulent wake of spheres is less at any given station in the wake but the growth rate remains fixed.

References

[1] Slattery, R. E. and Clay, W. G., "Turbulent wake of hypersonic bodies," ARS Preprint 2673-62 (1962).

[2] Demetriades, A. and Gold, H., "Transition to turbulence in the hypersonic wake of blunt-bluff bodies," ARS J. **32,** 1420–1421 (September 1962).

[3] Lyons, W. C., Jr., Brady, J. J., and Levensteins, Z. J., "Hypersonic drag, wake and stability data for cones and spheres," AIAA Preprint 64-44 (1964).

[4] Lees, L., "Hypersonic wakes and trails," AIAA J. **2,** 417–420 (1964).

[5] Slattery, R. E. and Clay, W. G., "Width of the turbulent trail behind a hypervelocity sphere," Phys. Fluids **4,** 1199 (1961).

[6] Ladenburg, R. W., Lewis, B., Pease, R. N., and Taylor, H. S., *Physical Measurementsin Gas Dynamics and Combustion* (Princeton University Press, Princeton, N. J., 1954).

[7] Feldman, S., "Trails of axi-symmetric hypersonic blunt bodies flying through the atmosphere," Research Rept. 82, Avco-Everett Research Lab. (December 1959).

[8] Labitt, M., "The measurement of electron density in the wake of a hypervelocity pellet over a six-magnitude range," TR 307, Massachusetts Institute of Technology Lincoln Lab. DDC 421762 (April 15, 1963).

[9] Labitt, M., "Measurement of the diameter of the electronic wake of the hypersonic pellets," TR 342, Massachussetts Institute of Technology Lincoln Laboratory, DDC 438816 (January 20, 1964).

[10] Schlichting, H., *Boundary Layer Theory* (McGraw-Hill Book Co. Inc., New York, 1960), p. 596.

[11] Kornegay, W. M., private communication, Lincoln Lab. Massachussetts Institute of Technology (July 1, 1960).